

A Model of the General Ocean Circulation Determined From a Joint Solution for the Earth's Gravity Field

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ABSTRACT

If the geoid and the satellite position are known accurately, satellite altimetry can be used to determine the geostrophic velocity of the surface ocean currents. The purpose of this investigation is to simultaneously estimate the sea surface topography, ζ , the model for the gravity field, and the satellite orbit. Satellite tracking data from fourteen satellites were used; along with Seasat and Geosat altimeter data as well as surface gravity data for the solution. The estimated model of ζ compares well at long wavelengths with the hydrographic model of ζ . Covariance studies show that the geoid is separable from ζ up to degree 9, at which point geoid error becomes comparable to the signal of ζ .

1. INTRODUCTION

The determination of the general ocean circulation is one of the more important applications future satellite altimeter missions will provide. If the height of the ocean surface relative to the geoid can be measured, the absolute geostrophic surface current velocity at a given location can be inferred. The Seasat and Geosat altimeter missions measured the height of the ocean to a precision suitable for this purpose [Tapley *et al.*, 1982; McConathy and Kilgus, 1987]. Errors in the gravity field have limited previous determinations of ζ for two reasons: 1. ζ is measured with respect to the geoid, an equipotential surface defined by the gravity field model, and 2. gravity field errors are the primary limitation in the accuracy of the satellite orbit computation.

The radar altimeter carried by Earth orbiting satellites measures the range between the radar antenna and the instantaneous ocean surface. If the geocentric position of the satellite is known, the altimeter measurement can be differenced from the computed satellite height to yield the ocean surface height at the sub-satellite point. If there were no forces acting except the Earth's gravity and centrifugal force, the ocean surface would coincide with an equipotential surface referred to as the geoid, N . However, a number of effects cause the height of the ocean surface to deviate from the geoid height. Among these, ocean currents cause deviations with maximum amplitudes of about one meter. If an accurate estimate of N is available, the measured height of the ocean surface (corrected for tides and other effects [Tapley *et al.*, 1982]) and the geoid height may be differenced to provide an estimate of ζ .

Previous maps of ζ with wavelengths comparable to a degree and order 6 spherical harmonic model (6000 km) have been determined using Seasat altimeter data [Tai and Wunsch, 1984; Engelis, 1986]. Each of these approaches adopted a mean sea surface computed using Seasat altimeter data which was then differenced with a low degree and order geoid to give an estimate of the long wavelength components of ζ . The error in the gravity field model has been a limiting factor in the accuracy achieved in these investigations.

The results described here were derived as an adjunct to a concentrated effort to determine an improved gravity field model for the Topex/Poseidon altimeter mission [Tapley *et al.*, 1987]. The use of altimeter data in a gravity field solution, without correcting for ζ , can allow the oceanographic signal due to ζ to be aliased into the gravity field. The potential for simultaneously improving the geoid and ζ using satellite altimetry, surface gravity observations, and hydrography has been discussed by Wunsch and Gaposchkin [1980]. Since the influence of the gravity field on the geoid and the orbit, and the effect of ζ , are both present in the altimeter measurement, a more rigorous treatment of the problem would involve estimating the parameters which define both models simultaneously. This investigation demonstrates a joint solution for ζ , the gravity field, and the satellite orbit using satellite altimeter and tracking data.

2. METHODS

Seasat and Geosat tracking data and altimeter data along with tracking data from twelve additional satellites and surface gravity data were processed in a simultaneous solution using a modified least squares estimator [Tapley *et al.*, 1987]. The estimated parameters included a set of spherical harmonic geopotential coefficients, complete to degree and order 50 with selected higher order terms, the spherical harmonic coefficients of ζ , complete to degree and order 15, and ocean tide parameters. Solar radiation pressure, atmospheric drag, and doppler tracking station coordinates were adjusted as individual satellite parameters. Altimeter biases and scale

factors for the significant wave height correction were estimated for Seasat and Geosat also. In addition to Seasat and Geosat altimeter data, the different types of satellite tracking data included laser range data, doppler data, optical data, and altimeter crossover data.

The stationary ocean surface is composed of the sum of N and ζ . The Earth's gravitational potential can be expressed in spherical harmonic form as:

$$U = \frac{\mu}{r} \left[1 + \sum_{l=2}^{L_{\max}} \sum_{m=0}^l \left(\frac{r_e}{r} \right)^l \bar{P}_{lm}(\sin\phi) [\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda] \right] \quad (1)$$

where \bar{C}_{lm} , \bar{S}_{lm} are the normalized spherical harmonic coefficients of degree l and order m , μ is the product of the gravitational constant and the total mass of the earth, r_e is the mean equatorial radius of the earth, r, ϕ, λ are the radial distance, geocentric latitude and longitude measured from the zero meridian, $\bar{P}_{lm}(\sin\phi)$ is the normalized associated Legendre function ($m \neq 0$) or the normalized Legendre polynomial of degree l ($m=0$), and L_{\max} is the truncation degree of the geopotential. The position vector (x, y, z) of the sub-satellite point, which defines the geoid height, is computed by iteratively solving [Shum, 1983]:

$$W(x, y, z) = W_0 \quad \text{and} \quad \nabla W / |\nabla W| = \hat{u}_h \quad (2)$$

where $W(x, y, z) = U + T$, the total potential at the sub-satellite point, T is the rotational potential, W_0 is value of the total potential corresponding to mean sea level, and \hat{u}_h is a unit vector normal to the geoid and passing through the satellite. Thus the set of constant parameters, \bar{C}_{lm} and \bar{S}_{lm} , in Equation (1) are common to both the satellite dynamics and the geoid definition (2). A spherical harmonic model was employed to represent the height of ζ as:

$$\zeta = \sum_{l=1}^{l_{\max}} \sum_{m=0}^l [C'_{lm} \cos m\lambda + S'_{lm} \sin m\lambda] \bar{P}_{lm}(\sin\phi) \quad (3)$$

where C'_{lm} , S'_{lm} are the surface spherical harmonic coefficients of the model for ζ . In this investigation, ζ is constrained to be zero over land areas and in polar regions ($|\phi| > 70^\circ$). While the altimeter data with the land constraints do not form an equally spaced data set, the estimate for the coefficients retains the general characteristics of a fully orthogonal parameter set.

Portions of the Seasat and Geosat altimeter data sets were selected to correspond to the time periods when the best ground tracking data was available. Using the Preliminary Topex Gravity Field (PTGF2) [Tapley et al., 1987], reference orbits were computed using tracking data and altimeter crossover data. In all, 30 days of Seasat altimeter data (7/28-8/15, 9/15-9/27, 1978) and 54 days of Geosat altimeter data (11/17/86-12/4/86, 12/21/86-1/24/87) were processed. The altimeter data were edited including the removal of data if the water depth was less than 2000 m (to remove data with large tidal uncertainties) or if the data was over an ocean trench or seamount. The data were corrected for the unmodeled short wavelength features of the geoid ($> \text{degree } 50$) and compressed to form altimeter measurements 10 seconds apart.

There was no *a priori* variance on the coefficients of ζ . The *a priori* variance of the geopotential coefficients, v_l^2 , were computed using Kaula's rule [Kaula, 1966] which can be stated as $v_l^2 = \alpha \cdot 10^{-10} l^{-4}$, where α is a scale factor equal to 0.04.

3. RESULTS

To evaluate the satellite altimeter solution for ζ , a spherical harmonic model of ζ was constructed from hydrographic data. The Levitus ocean surface referenced to 2250 dbars is adopted for this purpose [Levitus, 1982]. Grid points on land and in the polar regions ($|\phi| > 70^\circ$) were set to zero and a degree and order 15 spherical harmonic model was fit to the data using a minimum variance estimator. Using the estimated values for the spherical harmonic coefficients, ζ can be computed as a function of latitude and longitude using Equation (3). Figure 1 depicts maps of ζ computed from both the altimeter and Levitus derived spherical harmonic coefficients complete to degree and order 6. The major gyres of the ocean circulation are depicted in both solutions. Up to degree and order 6, the altimeter and hydrographic models of ζ compare well. The agreement between the hydrographic and altimetric results is better than in previous altimeter solutions for ζ , suggesting a significant improvement has been achieved by simultaneously solving for the gravity field, the satellite orbit, and ζ . Further details of the solution procedure and the results are given by Tapley et al., [1988].

The improvement in the geoid solution may be evaluated by computing new reference orbits (using the estimated models) and analyzing the residuals from the compressed altimeter data. It

was found that the altimeter residuals improved from 225 cm (for a gravity field without altimeter data) to 30 cm, indicating the geoid model has been substantially improved.

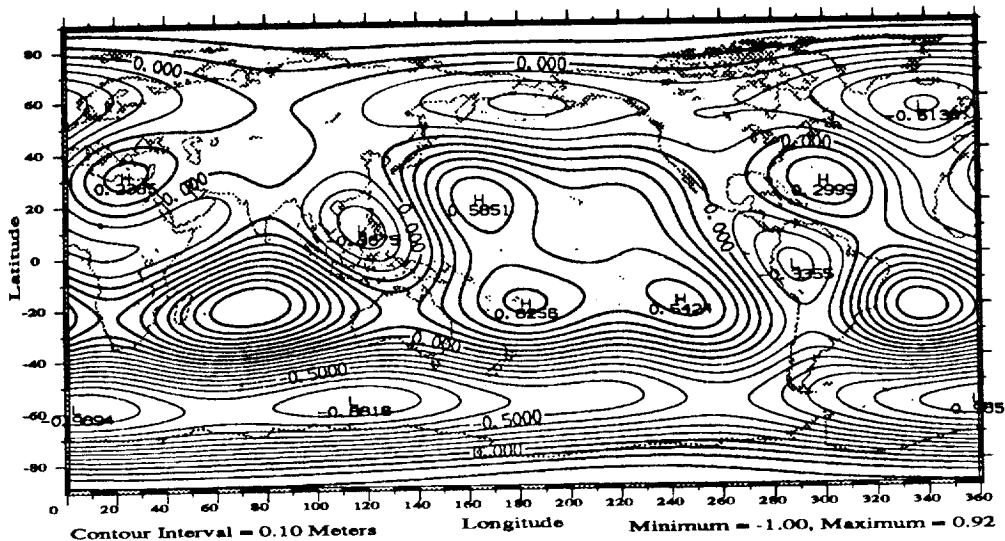


Figure 1a. The estimated model of ζ to degree and order 6

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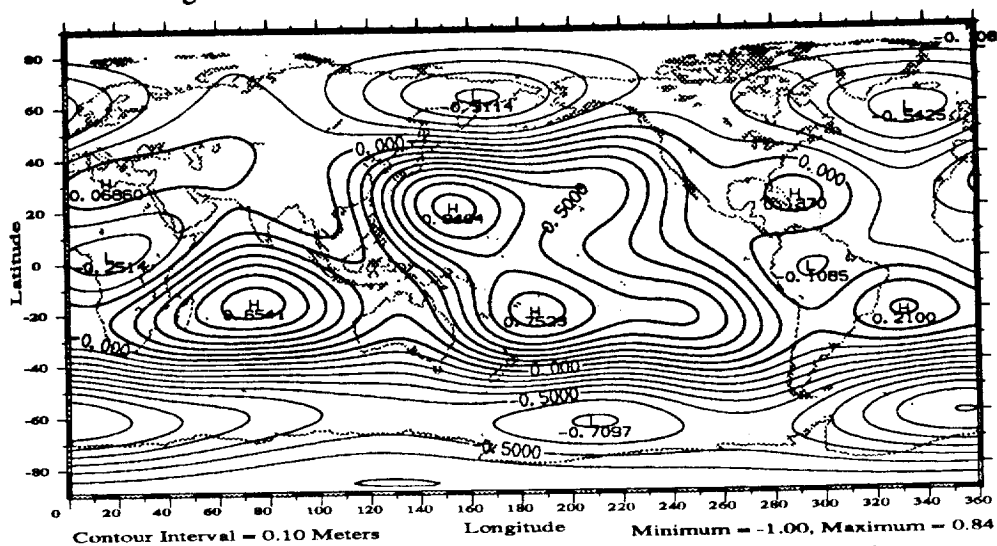


Figure 1b. The Levitus model of ζ to degree and order 6

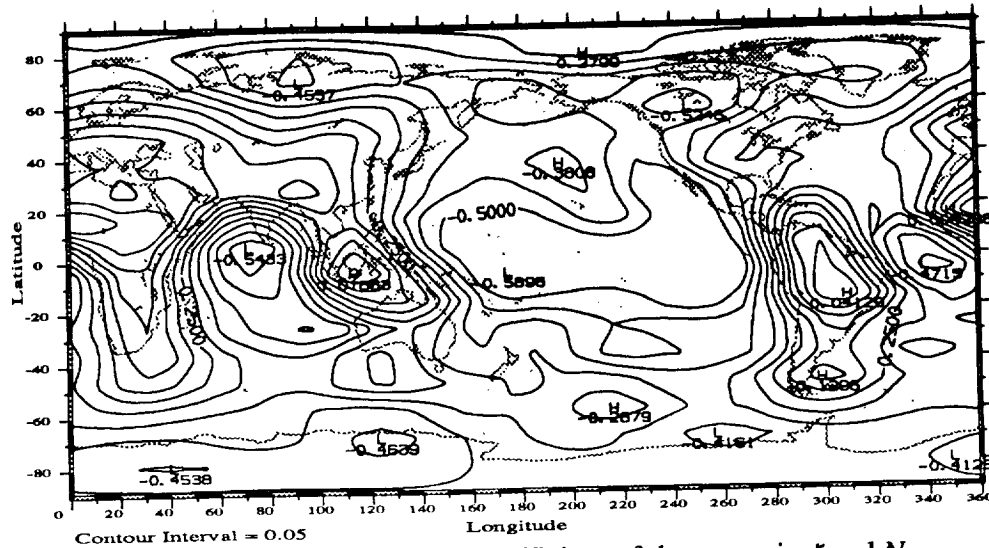
4. COVARIANCE ANALYSIS

One benefit of simultaneously estimating both the geopotential coefficients and the coefficients of ζ is that the resulting covariance matrix will describe not only the individual errors in N and ζ , but also the correlation of these errors. Considerable effort was expended to determine the relative weights of the data and the scaling of the covariance matrix.

By examining the degree variance of the ζ and the degree error variance of N (Table 1) it is seen that beyond degree 9, geoid error is larger than the signal of ζ . The covariance matrix for the coefficients of ζ and the geopotential can also be used to compute the standard deviation of the error in N and ζ . The error in N is largest over the land areas (where there is no altimeter data) and in equatorial regions. For terms up to degree 6, the geoid height is accurate to about 8 cm (Table 1). The error in ζ mimics the geoid error, the largest error occurs near the equator and increases with degree. The magnitude of the error in ζ is about 10 cm for terms up to degree 6 (Table 1). In addition to their individual errors, the correlation coefficient for the error in N and ζ can be analyzed to determine if the two quantities are separable. Figure 2 shows the geographical variation of the correlation coefficient for terms in the covariance matrix up to degree 6 in both N and ζ . This figure verifies that the solution for N and ζ are separable, at this degree, since the correlation coefficients are less than 0.6 globally.

Table 1. The degree variance of ζ and the degree error variance of N and ζ

Degree	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Signal of ζ (cm)	10	20	11	13	14	17	11	10	12	12	16	12	14	11	7
Error in N (cm)	<1	1	2	4	4	6	8	10	11	14	13	13	16	14	16
Error in ζ (cm)	5	3	4	4	5	5	7	8	9	9	11	11	11	9	9


Figure 2. The correlation coefficient of the errors in ζ and N

5. CONCLUSIONS

It has been shown that a global estimate of ζ can be obtained from a joint solution for ζ and the gravity field using multiple satellite tracking data and altimeter data. The simultaneous solution for the ζ , the gravity field, the satellite orbit, and other geophysical parameters provides a substantially improved estimate for the gravity field and the long-wavelength signal of the general ocean circulation. This estimate of ζ compares favorably at long wavelengths with hydrographic determinations of ζ . The accuracy of the solution decreases as shorter wavelength features are examined, primarily due to the influence of geoid error.

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